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Ohmic Dissipative MHD Pressure-driven Coupled-flow and Heat Transfer Across a Porous Medium with Thermal Radiation

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Authors' contributions

This work was carried out in collaboration between all authors. Author JFB formulated the Mathematical model of the problem, author MAM performed the mathematical analysis and author OAE wrote the first draft of the manuscript. Authors TOO and OEE managed the analyses of the study and the literature searches. All authors read and approved the final manuscript.

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ABSTRACT

In this study, a pressure driven flow of a magnetohydrodynamic steady coupled-flow across a porous layer horizontal bottom plate with buoyancy force is investigated. The heat transfer problem is also examined by taking viscous and Ohmic dissipation and radiation effects in the energy equation into consideration. The velocity and temperature slip boundary conditions are taken at the plate and at the interface of the porous medium and clear fluid, it is assumed that velocity components to be continuous and the jump in shearing stresses. The solutions to the problem are

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obtained by employing fourth order Runge-Kutta scheme along with shooting technique and the effects of the pertinent parameters entrenches in the flow system are shown graphically and quantitatively discussed. The results shows that an increase in the thermal convection and pressure gradient enhances the flow rate in both region but the effect was great at the clear region than the porous medium region.

Keywords: Porosity; thermal radiation; pressure gradient; magnetohydrodynamic flow; dissipation.

2010 Mathematics Subject Classification: 53C25; 83C05; 57N16.

1 INTRODUCTION

Theoretical study of the electrically conducting fluid flow in the presence of magnetic field has been considered by several scholars under different configurations and conditions. It has many usefulness in the areas of science and technology which includes geophysics, astrophysics, etc. The results of such investigation is adopt by the engineers in the construction of magnetohydrodynamic (MHD) controlled fusion, MHD power generator, heat exchangers and MHD pumps. Flow past a plate has numerous applications in the industry. In Stokes [1], the analytical solution of viscous incompressible fluid of Navier-Stokes equation for the flow past a horizontal plate was examined while Turbatu et al. [2] presented new results to the second Stokes problem by considering rising/diminishing oscillating momentum. The thermal influences are reported by Puri and Kythe [3] on the second Stokes problem.

For non-Newtonian fluids, the second problem of Stokes was studied by high numbers of researchers, among them is Ai and Vafai [4], Hayat et al. [5], Nazar et al. [6] and Fetecau et al. [7]. The flow past a permeable medium is gaining attention in nature. The first and second problems of Stokes in a permeable medium in the presence of magnetic field have been considered by Chauhan and Soni [8], Salawu and Dada [9], Sacheti and Bhatt [10] and Salawu and Fatunmbi [11]. The study of temperature transfer under a porous substrate medium in gaining prominent in many engineering and scientific applications. Vafai and Kim [12], Salawu and Amoo [13] carried out study on convection problem involving a porous substrate medium in a plate. The numerical studied was done by the authors. Huang and Vafai [14] verified the same problem by an integral approach and obtained a semi-analytical solution for the momentum and heat equations. Nield and Kuznetsov [15] examined the boundary layer of the same problem, and noticed that the heat flux rises as the porosity values increased. Chauhan and Olkha [16], Dada and Salawu [17] and Kareem et al. [18] investigated energy transfer in a fluid flow through a porous medium in a moving sheet. It is exciting to examining heat transfer impacts when the flow past a porous substrate is stimulated by pressure and buoyancy force with plate motion in the presence magnetic field because it occur basically in engineering applications situations. Also, it can serve as an estimate for complex situations natural phenomenons when a clear fluid moves adjacent to a fluid-saturated porous medium with matching conditions at the interface. At the clear fluid-porous medium interface matching conditions occur as reported by Ochoa-Tapia and Whitaker [19,20]. Matching equations needs flow velocity continuity but a discontinuity at the interface shear stress. The study with different types of momentum and heat slip, radiation, dissipation and magnetic effects boundary conditions are important in thermal insulation, filtration process, oil recovery enhancement process, ceramic processing, heat exchangers, and in chemical engineering. Most of the existing work relating to the present study was done without considering the combined effect of pressure gradient and the thermal Grashof number as well as the thermal radiation. The parameters have great influences on the rate of fluid flow and heat transfer within the a system.

In the present study, pressure-driven MHD steady coupled-flow is examined past a porous medium with impermeable bottom plate. Heat transfer associated with the problem is also investigated taking radiation, Ohmic and viscous dissipation effects in the energy component. Momentum and heat boundary conditions are taken at the plate. The convective MHD flow and temperature transfer at the porous medium interface and clear fluid are assumed to be jump in shearing stresses and continuous. The computational solutions to the problem is obtained by using Nachtsheim-Swigert shooting technique coupled with sixth-order Runge-Kutta and the effects of the pertinent parameters embedded in the flow are shown graphically and discussed.

2 FORMULATION OF THE PROBLEM

Consider a steady flow of a viscous, incompressible and electrically conducting fluid past an infinite horizontal impermeable plate at the bottom of a porous layer of thickness h. A Cartesian coordinate system is assumed where x-axis is taken along the infinite horizontal plate, and the y-axis is taken normal to the plate. The fluid fills the semi-infinite region $y \geq 0$, and the porous layer $(-h \leq y < 0)$.

Initially, the fluid and the bottom plate of the fluid-saturated porous layer are stationary and kept at the same constant temperature T_∞. The bottom impermeable plate is given its own plane with a velocity U_0, and the temperature of the plate is raised to T_w at the same time. Also a magnetic field of uniform strength B_0 is applied normal to the bottom plate. The magnetic Reynolds number is assumed to be small and hence the induced magnetic field is neglected. Both velocity and temperature slips are assumed at the bottom plate. Thermal radiation is also taken into account. It is assumed that the medium is optically thin with relatively low density. The expression of the radiative heat flux following Cogley et al.[21] is taken as

$$\frac{\partial q}{\partial y} = 4(T - T_\infty) \int_0^\infty K_{\lambda w} \left(\frac{\partial e_{b\lambda}}{\partial T}\right)_w d\lambda = 4I^*(T - T_\infty) \tag{2.1}$$

where $K_{\lambda w}$ is the absorption coefficient at the wall and $e_{b\lambda}$ is the plank function. For this optically thin limit, the fluid absorbs radiation emitted by boundaries other than its own emissions. At higher temperatures, such as in nuclear engineering and space technology applications, radiation effects are quite significant and the presence of magnetic field also plays an important role. The plate and the porous layer are infinite in extent so all the flow variables are independent of x, and so their derivatives with respect to x vanish. Only non-zero velocity component

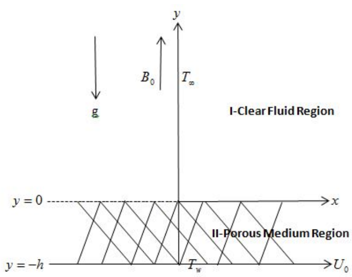

Fig. 1. The formulation geometry

For clear fluid region $(y \geq 0)$

$$-\frac{dP}{dx_1} + \mu\left(\frac{d^2u_1}{dy^2}\right) - \sigma B_0^2 u_1 + g\beta(T_1 - T_\infty) = 0 \tag{2.2}$$

$$K\left(\frac{d^2T_1}{dy^2}\right) + \mu\left(\frac{du_1}{dy}\right)^2 + \sigma B_0^2 u_1^2 - 4I*(T_1 - T_\infty) = 0 \tag{2.3}$$

For porous medium region $(-h \leq y < 0)$

$$-\frac{dP}{dx_1} + \mu\left(\frac{d^2u_2}{dy^2}\right) - \frac{\mu}{K_0}u_2 - \sigma B_0^2 u_2 + g\beta(T_2 - T_\infty) = 0 \tag{2.4}$$

$$K\left(\frac{d^2T_2}{dy^2}\right)+\frac{\mu}{K_0}u_2^2+\mu\left(\frac{du_2}{dy}\right)^2+\sigma B_0^2u_2^2-4I*(T_2-T_\infty)=0 \quad (2.5)$$

The corresponding boundary conditions are as follow

$$\begin{aligned} &u_1=u_2=0,\ T_1=T_2=T_w,\ \ for\ \ y=0\\ &u_1=u_2=U_0=-h,\ T_1=T_2=T_\infty,\ \ as\ \ y=\infty \end{aligned} \quad (2.6)$$

where u_1 and u_2 are the velocity in the clear and unclear region, T_1 and T_2 are the fluid temperature in the clear and unclear region. B_0 is the magnetic field strength, g is the gravity, K_0 is the porosity parameter. The physical quantities $\mu,\ \rho,\ \sigma,\ Cp,\ K,\ I$ and β are the coefficient of viscosity, density, electric conductivity of the fluid, specific heat at constant pressure, thermal conductivity, radiation parameter and thermal expansion coefficient.

Introducing the following dimensionless quantities into equations (2.2)-(2.6)

$$\eta=\frac{y}{h},\ x=\frac{x_1}{h},\ u=\frac{u_1}{u_0},\ p=\frac{Ph}{\mu u_0}, \quad (2.7)$$

$$\phi=\frac{T_1-T_\infty}{T_w-T_\infty},\ \theta=\frac{T_2-T_\infty}{T_w-T_\infty},\ U=\frac{u_2}{u_0} \quad (2.8)$$

The governing equations (2.1)-(2.6) reduce to the dimensionless form:

$$G+\frac{d^2u}{d\eta^2}-M^2u+Gr\phi=0 \quad (2.9)$$

$$\frac{d^2\phi}{d\eta^2}+Ec\left(\frac{du}{d\eta}\right)^2+EcM^2u^2-F\phi=0 \quad (2.10)$$

$$G+\frac{d^2U}{dx^2}-\left(\frac{1}{\lambda}+M^2\right)U+Gr\theta=0 \quad (2.11)$$

$$\frac{d^2\theta}{d\eta^2}+Ec\left(\frac{dU}{d\eta}\right)^2+Ec\left(\frac{1}{\lambda}+M^2\right)U^2-F\theta=0 \quad (2.12)$$

The corresponding boundary conditions becomes

$$\begin{aligned} &u=U=0,\ \ \phi=\theta=1,\ \ as\ \ \eta=0\\ &u=U=1,\ \ \phi=\theta=0,\ \ as\ \ \eta\to\infty \end{aligned} \quad (2.13)$$

where $M^2=\frac{h^2\sigma B_0^2}{\mu}$ is the magnetic parameter, $Gr=\frac{h^2g\beta}{\mu u_0}(T_w-T_\infty)$ is the thermal Grashof number $G=-\frac{dp}{dx}$ is the pressure, $Ec=\frac{\mu u_0^2}{K(T_w-T_\infty)}$ is the Eckert number, $F=\frac{4h^2I^*}{K}$ is the heat radiation parameter, $\lambda=\frac{K_0}{h^2}$ is the porosity parameter.

The physical parameters of interest for this flow are the local skin friction C_f and the Nusselt number N_u given respectively as:

$$C_f=\frac{\tau_w}{\rho}u_0^2,\quad Nu=\frac{xq_w}{K(T_w-T_\infty)} \quad (2.14)$$

where k is the thermal conductivity of the fluid, τ_w and q_w are respectively given by

$$\tau_w=\mu\left(\frac{du}{d\eta}\right)_{\eta=0},\quad q_w=-K\left(\frac{d\phi}{d\eta}\right)_{\eta=0} \quad (2.15)$$

$$Re_x^{\frac{1}{2}}C_f=u^{'}(0) \quad (2.16)$$

$$Nu_xRe_x^{-\frac{1}{2}}=\phi^{'}(0) \quad (2.17)$$

where $Re_x=\frac{u_0x\rho}{\mu}$ is the Reynolds number.

3 RESULTS AND DISCUSSION

Since equations (2.9)-(2.12) satisfying equation (2.13) is highly nonlinear and it is a boundary value problem. It is solved numerically by applying Nachtsheim-Swigert shooting iteration technique along with Runge-Kutta sixth-order integration method. From the process of numerical computation, the skin-friction coefficient and the local Nusselt number, which are respectively proportional to $u'(0)$, $-\phi'(0)$. The computations have been performed by a program which uses a symbolic and computational computer language MAPLE 18.

Numerical calculations were made for different values of the parameters controlling the fluid dynamics in the flow regime for the two cases under consideration, i.e the clear and porous medium region with the following default value $G=0.5, M=2, Gr=0.5, \lambda=0.2, Ec=0.3, F=2$ except otherwise specify on the appropriate graph.

From Fig. 2, the effect of pressure G and magnetic field parameter M on the fluid flow velocity is examined while fixing other fluid flow parameters. It is established that increasing the parameter G increases the flow rate at the clear fluid region while M step down the rate of flow at the porous medium region as a result of increase in the fluid bonding force.

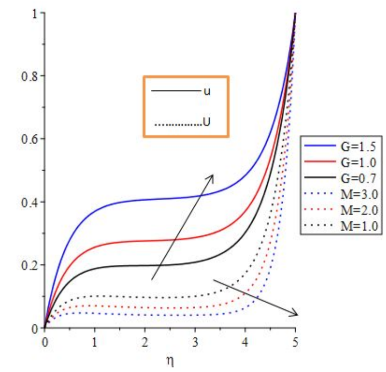

Fig. 2. Velocity profile for different values of G and M

Fig. 3 discusses the impact of thermal Grashof number Gr and magnetic field parameter M on the velocity profiles. Increase in the thermal Grsahof Gr reduces the fluid bonding forces and thereby causes free flow of the fluid but increase in the magnetic field M parameter decreases the momentum distribution of the flow by introducing a drag force called Lorentz force into the flow that resulted into diminishing the velocity profile as observed in Fig. 2.

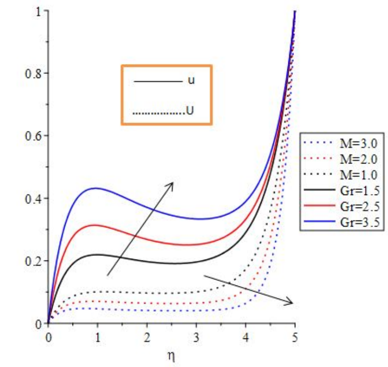

Fig. 3. Velocity profile for different values of Gr and M

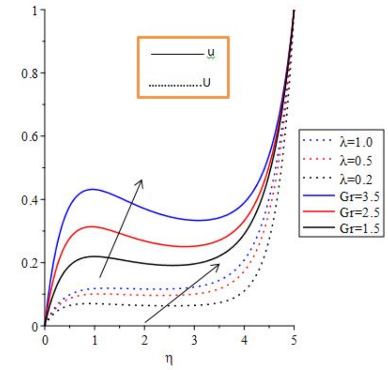

Fig. 4. Velocity profile for different values of Gr and λ

The reaction of fluid velocity to varying in the Grashof number Gr and Porosity term λ are displays in Fig. 4. A rise in the parameter Gr enhances the flow rate as shown in Fig. 3 while an increase in the inverse of the parameter λ increases the flow rate as it reduces the flow resistance in the flow regime. Hence, both parameters boosted the fluid momentum field.

From Fig. 5, the response of the flow momentum to difference values of the pressure gradient G and the Eckert number Ec are portrays in the figure. The plot shows that the pressure gradient propelled the flow distributions but the Eckert number Ec demonstrates little or no significant effect on the flow profiles. Eckert number describes the ratio of kinetic energy of the flow to the boundary layer enthalpy difference.

Fig. 6 shows the influences of the magnetic field M and the porosity parameter λ on the fluid flow rate in the system. The magnetic field parameter drag down the flow rate at the clear region of the system but the inverse of porosity parameter magnifies the flow rate due to an increase in the flow and thermal boundary layers that reduces the amount of heat that flow out of the system and thereby increases the collision rate of the fluid particles which then resulted to an increase in the velocity field.

The effect of pressure gradient and magnetic field parameters on the temperature profiles are illustrated in the diagram 7. It is noticed from the plot that an increase in the pressure gradient causes a rise in the heat distributions within the flow clear region while the magnetic field does not show any variation at the early stage of the heat distributions but as $\eta \to \infty$ the magnetic field started diminishes the temperature distributions rate in the porous medium region of the flow as seen in the plot.

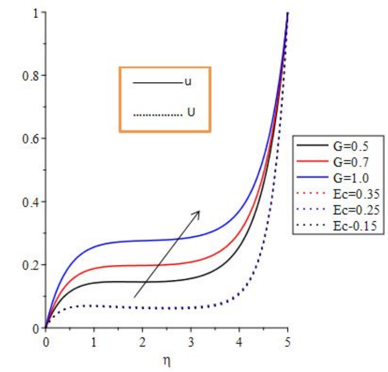

Fig. 5. Velocity profile for different values of Ec and G

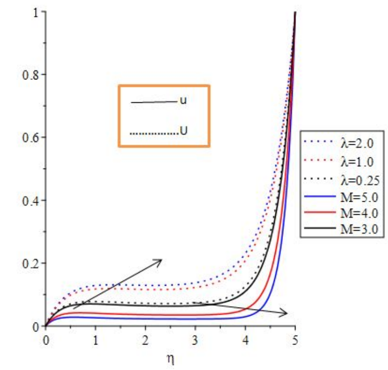

Fig. 6. Velocity profile for different values of G and λ

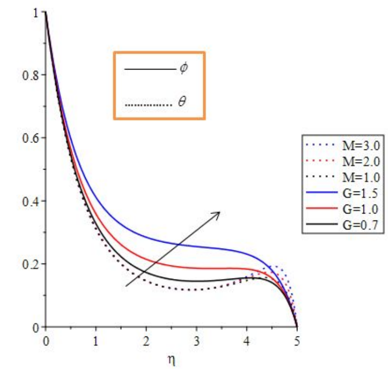

Fig. 7. Temperature profile for different values of G and M

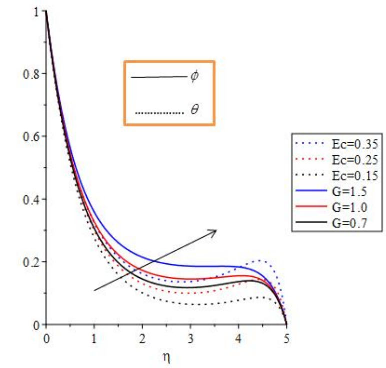

Fig. 8. Temperature profile for different values of G and Ec

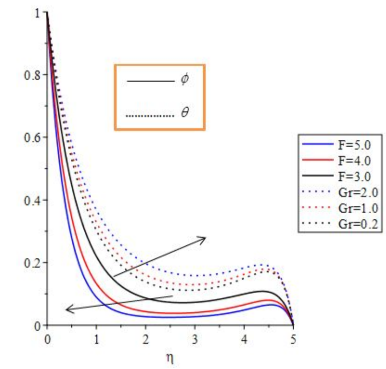

Fig. 9. Temperature profile for different values of F and Gr

Fig. 8 demonstrates the reaction of the temperature field to variation in the Eckert number in the unclear flow region and pressure gradient in the flow clear region. It is observed that the parameters enhances distributions of energy in both flow regions because the thermal boundary layer gets thicker as the parameter values increases which causes an increase in the amount of heat within the system, thereby influence the temperature profiles.

An enhancement in the temperature distributions is noticed in Fig. 9 as the thermal Grashof number parameter rises. This is expected since thermal buoyancy force break down the fluid molecular force which resulted in high collision rate among the fluid particles and thereby increases the temperature field in the system. But variational rise in the thermal radiation parameters decreases heat distribution rate within the flow region because energy evolved out of the system due to reduction in the thermal boundary layers that enable heat to escape from the clear flow regime to the environment.

Fig. 10 shows the influences of pressure gradient G and magnetic field M on the shear stress rate. An early increase in the effect of the parameter G is noticed which later decreases as it move far away from the wall while the parameter M decreases the shear stress rate near the plate but enhances the shear stress rate further away from the plate. The parameter Gr exhibited the same behavour as parameter parameter G while parameter λ also shows the same behavour with parameter M respectively as illustrated in Fig. 11.

Fig. 12 demonstrates the reaction of parameters F and Gr on the temperature gradient. It is noticed from the graph that an increase in the parameter F increases the heat gradient near the plate but reduces gradually as the heat distributes away from the wall. Also, the parameter λ shows a slight effect on the nusselt number. It reduces the temperature at the wall slightly and shows no variation as it moves far from the plate.

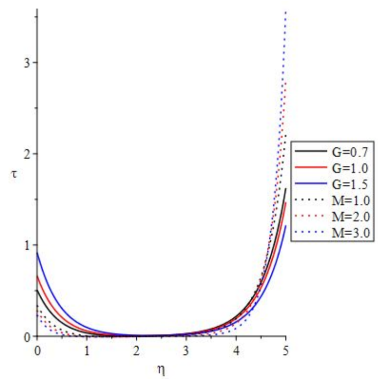

Fig. 10. Skin friction for different values of G and M

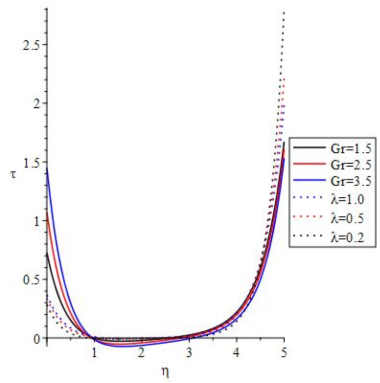

Fig. 11. Skin friction for different values of G and λ

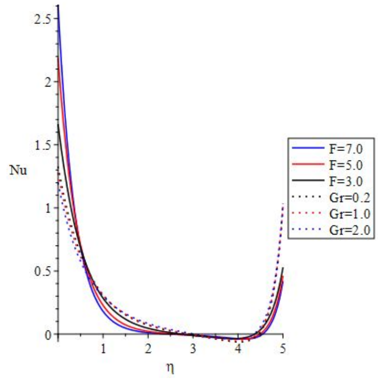

Fig. 12. Nusselt number profiles for different values of G and Ec

4 CONCLUSIONS

A computational solution is gotten to study the heat transfer effect on the flow through and across a porous layer due to an impermeable plate at its bottom. The radiative and ohmic dissipative flow is stimulated by pressure gradient and buoyancy force in the present analysis. The conclusions of the study are as follows:

The velocity in both regions increases when the pressure G and Grashof number Gr increases. However the magnetic parameter M or the inverse porosity parameter λ causes a decrease in the velocity.

The temperature enhances when the radiation and viscous dissipation parameters increases. It also increases by pressure gradient G and thermal Grashof number Gr.

The physical quantities of interest are the shear stress τ and the rate of heat transfer at the plate Nu. The parameters G and Gr rises the skin friction while the parameters M and λ diminishes the wall friction while the parameter F causes rises in the heat gradient.

The present analysis has applications in space technology, heat exchangers, ceramic processing, petroleum and nuclear engineering applications.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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