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Modelling of Daily Nigerian Naira - British Pound Exchange Rates Using SARIMA Methods

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Author's contribution

This work was carried out by author EHE. Author EHE designed the study, performed the statistical analysis, wrote the protocol, and wrote the first draft of the manuscript. Author EHE managed the analyses of the study, managed the literature searches and read and approved the final manuscript.

Research Article

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ABSTRACT

Aims: To fit a time series model to daily Naira-Pound exchange rate series.
Study Design: Seasonal Autoregressive Integrated Moving Average Model.
Place and Duration of Study: Department of Mathematics/Computer Science, Rivers State University of Science and Technology, Nigeria, from December 2012 to March 2013.
Methodology: The correlogram of a non-seasonal difference of the 7-point difference of the data was plotted. On the basis of that plot, a seasonal autoregressive integrated moving average $(0, 1, 1)x(0, 1, 1)_7$ model was proposed and fitted. This model was compared with a suggestive ARIMA model with a view to establishing SARIMA supremacy.
Results: Seasonality of order 7 is evident from the analysis of the differences of the seasonal differences of the original series. All three moving average parameters (i.e. for lags 1, 7 and 8) of the SARIMA model are highly significant, their P-values being 0.0005, 0.0000 and 0.0001 respectively. The model agrees very closely with the observed data. Up to 51% of variations in the data set are explained by the model. The residuals are observed not to be correlated with each other. On the other hand only 8% of the variability in the data set is accounted for by the ARIMA(1, 1, 1) model.
Conclusion: The SARIMA model more adequately represents the data set.

Keywords: Daily Naira-Pound exchange rates; SARIMA model; Nigeria.

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1. INTRODUCTION

Time series modeling of foreign exchange rates has engaged the attention of many researchers. A few of such researchers are Onasanya and Oyebimpe [1], Appiah and Adetunde [2], Etuk [3,4,5], Etuk and Igbudu [6], etc. Many economic and financial time series are known to be seasonal as well as volatile. Seasonal autoregressive integrated (SARIMA) models were proposed by Box and Jenkins [7] to specifically model seasonal time series. Such a modeling approach shall be used to explain the variation in the daily exchange rates of the naira and the pound in this work. Etuk and Igbudu [6] fitted a $(0, 1, 0)x(2, 1, 1)_{12}$ SARIMA model to the monthly rates.

SARIMA modeling has been discussed and applied extensively in the literature. A few of the authors involved are Madsen [8], Surhatono [9], Saz [10]. Expectedly, this modeling procedure compares favourably with other techniques and even often exhibits remarkable advantage over them. The comparative advantage of the fitted SARIMA model over an adequate ARIMA model shall be demonstrated.

2. MATERIALS AND METHODS

The data for this work are 113 daily Naira/Pound exchange rates from 8th December 2012 to 30th March 2013 published in the Nation newspaper in the website www.thenationonlineng.net.

2.1 SARIMA Modelling

A stationary time series $\{X_t\}$ is said to follow an autoregressive moving average model of order p and q denoted by ARMA(p, q) if

$$X_t - \alpha_1 X_{t-1} - \alpha_2 X_{t-2} - \dots - \alpha_p X_{t-p} = \varepsilon_t + \beta_1 \varepsilon_{t-1} + \beta_2 \varepsilon_{t-2} + \dots + \beta_q \varepsilon_{t-q} \qquad (1)$$

or

$$A(L)X_t = B(L)\varepsilon_t \qquad (2)$$

where $\{\varepsilon_t\}$ is a white noise process and the α's and β's are constants.

$$A(L) = 1 - \alpha_1 L - \alpha_2 L^2 - \dots - \alpha_P L^P$$

$$B(L) = 1 + \beta_1 L + \beta_2 L^2 + \dots + \beta_q L^q$$

and L is the backward shift operator defined by $L^k X_t = X_{t-k}$. For stationarity and invertibility the roots of A(L) = 0 and B(L) = 0 must all be outside the unit circle respectively.

Suppose $\{X_t\}$ is non-stationary and non-seasonal, Box and Jenkins [7] proposed that if it is differenced a certain number of times it might become stationary. Let ∇X_t be the first difference of X_t. Then $\nabla = 1 - L$. Let d be the minimum number of times necessary for $\nabla^d X_t$ the d^{th} difference of $\{X_t\}$ to be stationary. If $\{\nabla^d X_t\}$ follows an ARMA(p,q) model as in (1) we say that $\{X_t\}$ follows an autoregressive integrated moving average model of orders p, d and q, designated ARIMA(p, d, q).

Suppose $\{X_t\}$ is seasonal with period s, Box and Jenkins[7] proposed that seasonal (i.e. s-lag) differencing (combined with some non-seasonal differencing, if necessary) could render the series stationary. Let d and D be the (minimum) degrees of non-seasonal and seasonal differencing necessary, respectively, to make the series stationary. $\{X_t\}$ is said to follow a (p, d, q)x(P, D, Q)$_s$ seasonal autoregressive integrated moving average (SARIMA) model if

$$A(L)\Phi(L^s)\nabla^d\nabla_s^D X_t = B(L)\Theta(L^s)\varepsilon_t$$

where $\Phi(L) = 1 + \phi_1 L + \phi_2 L^2 + \ldots + \phi_P L^P$ and $\Theta(L) = 1 + \theta_1 L + \theta_2 L^2 + \ldots + \theta_Q L^Q$ are the seasonal autoregressive and moving average operators respectively and the ϕ's and θ's constants such that stationarity and invertibility are guaranteed. Moreover $\nabla_s = 1 - L^s$.

2.2 Model Estimation

2.2.1 Order Determination

Preliminary data analysis employs graphical and tabular methods. The differencing orders d and D are determined as the observed minimum orders to which non-seasonal and seasonal differencing are done for stationarity to be attained respectively. The time plots of the series and the differences (if necessary) are the basis for estimation of the orders. Generally non-seasonal differencing of the seasonal difference of the series yields a stationary series. That is, d = D = 1. Non-stationarity shall be tested by the Augmented Dickey-Fuller unit root test.

If the autocorrelation function (ACF) of the differenced series has significant spikes at lags which are multiples of s, then s is the period of seasonality. If this spike is negative it suggests the existence of a seasonal moving average (MA) component. If positive it is an indication of the involvement of a seasonal autoregressive (AR) component.

The nonseasonal AR order p is estimated as the cut-off lag of the partial autocorrelation function (PACF). Its MA counterpart is estimated by the cut-off lag of the autocorrelation function (ACF). Similarly P and Q, the seasonal AR and MA orders respectively are estimated.

In particular a (0, 1, 1)x(0, 1, 1)$_S$ SARIMA model is suggestive if the ACF has significant lags at lags 1, s and s+1, with the spikes at lags s-1 and s+1 comparable, Box and Jenkins [7].

2.2.2 Parameter Estimation

After order determination, the model parameters are estimated. Involvement of items of white noise in the model calls for the use of non-linear optimization techniques. An initial estimate of the parameter in question is usually made. This estimate is improved upon sequentially by an iterative process until there is convergence to an acceptable estimate depending on the specified error margin. Traditionally the least squares procedure, the maximum likelihood procedure, the maximum entropy techniques are a few of the approaches that are applicable.

Linear optimization algorithms have been proposed for ARMA modeling. For pure AR and MA processes such linear techniques are in existence (see, for example Box and Jenkins [7] ,Oyetunji [11]). Attempts have been made to propose linear techniques for the mixed ARMA models (see Etuk [12], for instance).

However for this work the software Eviews which is based on the least error sum of squares approach shall be used.

2.2.3 Diagnostic Checking

Fitted models need to be tested for goodness-of-fit to the data. Residuals of a fitted model must be analyzed for that purpose. If the residuals are uncorrelated and normally distributed with zero mean, this is an indication of model adequacy.

3. RESULTS AND DISCUSSION

The time plot of the series DNPER in Fig. 1 shows a generally downward trend. Non-seasonal differencing yields a series DDNPER. The time-plot of DDNPER in Fig. 2 shows a horizontal trend. The augmented Dickey-Fuller tests summarized in Table 1 reveal that DNPER is non-stationary whereas DDNPER is stationary. Seasonal (i.e. 7-point) differencing produces a series SDDNPER with a generally slightly negative trend (Fig. 3). Nonseasonal differencing of SDDNPER yields the series DSDDNPER with a horizontal trend (Fig. 4). The unit-root tests of Table 1 show that SDDNPER and DSDNPER are non-stationary and stationary respectively especially at 1% level. The correlogram of DDNPER in Fig. 5 shows a significant spike at lag 1 for both the ACF and the PACF suggesting an ARIMA(1, 1, 1) model which is estimated as summarized in Table 2 as:

$$\underset{}{DDNPER_t} - \underset{(\pm 0.3173)}{0.0639} DDNPER_{t-1} + \underset{(\pm 0.2980)}{0.3551}\varepsilon_{t-1} = \varepsilon_t \qquad (3)$$

The correlogram of DSDDNPER in Fig. 6 shows an ACF with significant spikes at lags 1, 6 and 7. The negative sign of the autocorrelation at the significant lag 7 spike is an indication of the involvement of a seasonal MA component, of order one. It is noted that though the spike at lag 8 is not significant it is positive as that at lag 6. This is an evidence of a (0, 1, 1)x(0, 1, 1)$_7$ SARIMA model. That means that DSDDNPER follows the model

$$DSDDNPER_t = \varepsilon_t + \beta_1\varepsilon_{t-1} + \beta_7\varepsilon_{t-7} + \beta_8\varepsilon_{t-8}$$

which is estimated in Table 3 as

$$DSDDNPER_t = \varepsilon_t - \underset{(\pm 0.0812)}{0.2899}\varepsilon_{t-1} - \underset{(\pm 0.0002)}{0.8561}\varepsilon_{t-7} + \underset{(\pm 0.0838)}{0.3488}\varepsilon_{t-8} \qquad (4)$$

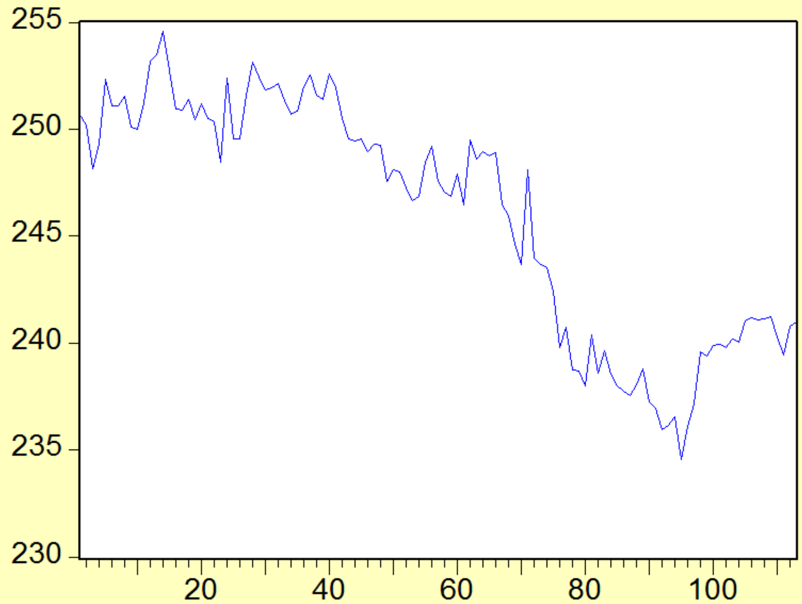


FIG 1: Time Plot of DNPER

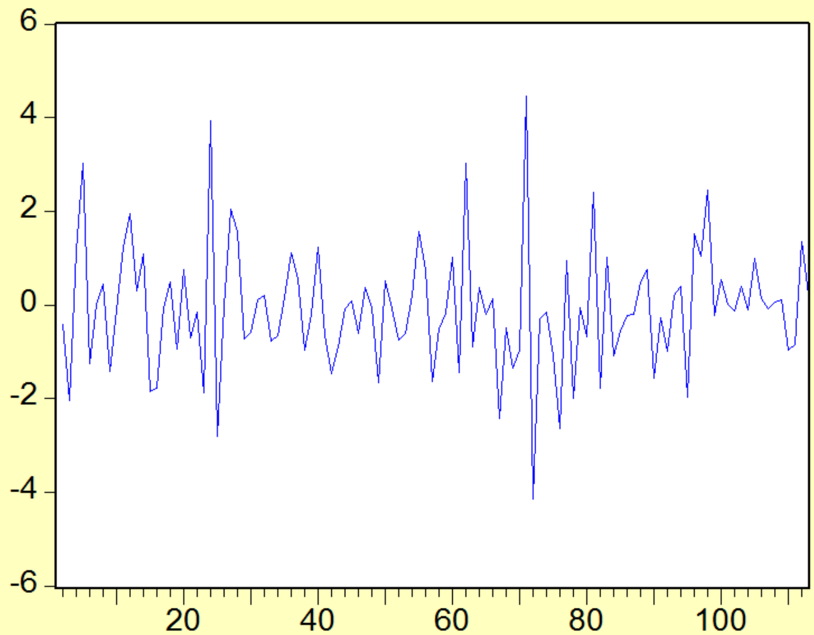


FIG 2: Time Plot of DDNPER

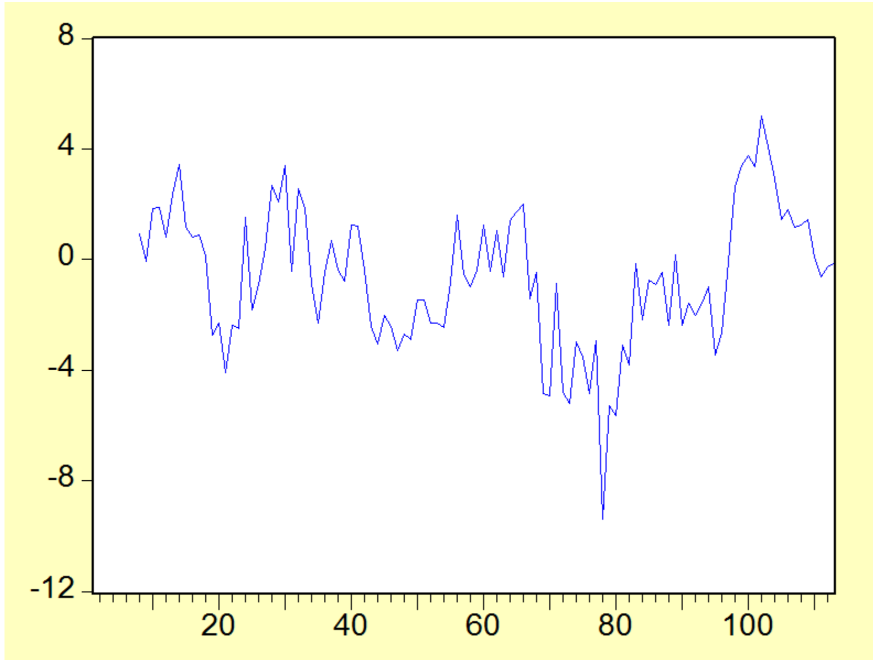


FIG 3: Time Plot of SDDNPER

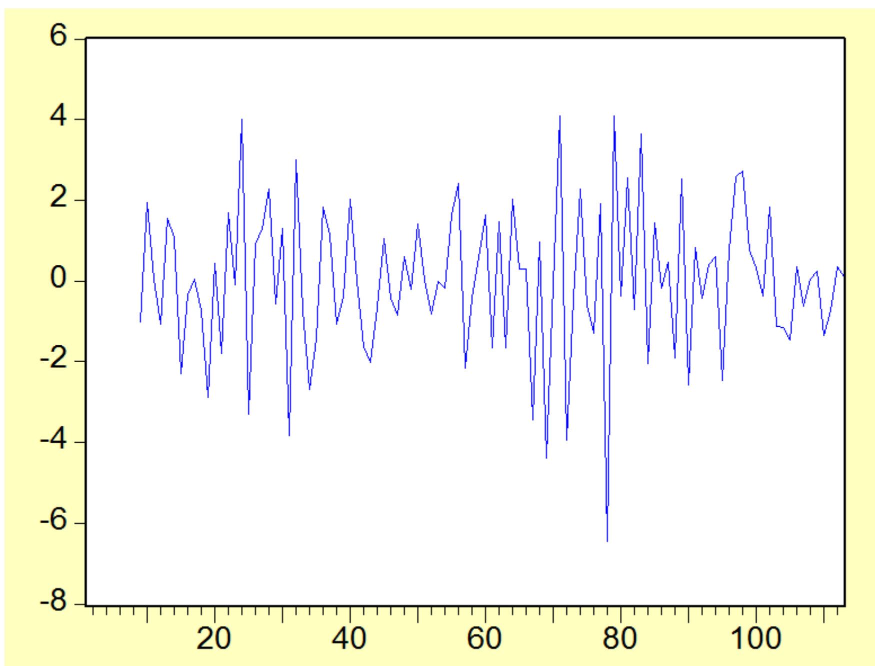
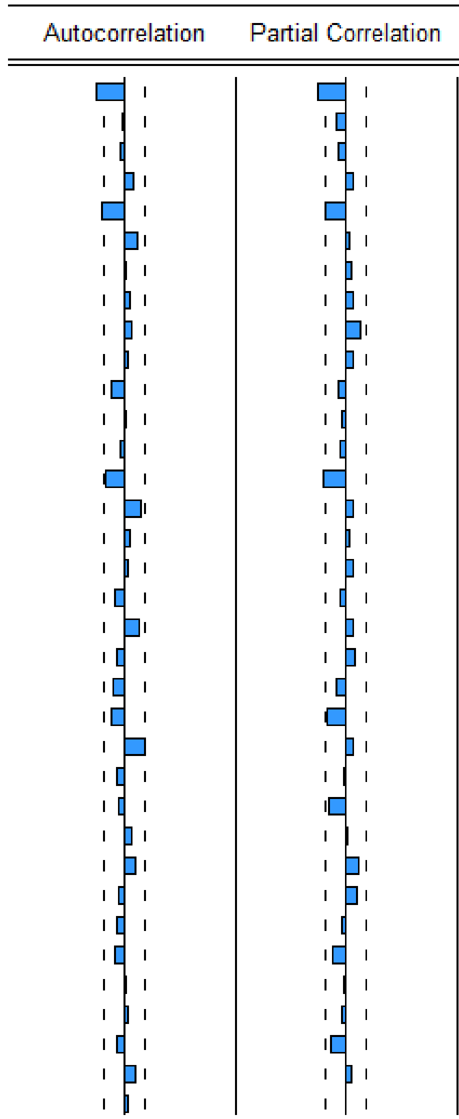


Fig 4:Time Plot of DSDDNPER

Table 1. Augmented dickey fuller tests for non-stationarity

Series	Test statistic	1% Critical value	5% Critical value	10% Critical value	Conclusion
Dnper	-0.9371190	-3.4917	-2.8882	-2.5808	Non-stationary
ddnper	-5.686711	-3.4922	-2.8884	-2.5809	Stationary
sddnper	-3.125153	-3.4959	-2.8900	-2.5818	Non-statonary*
dsddnper	-5.148459	-3.4965	-2.8903	-2.5819	stationary

**Test significant at 5%level but non-significant at 1% level.*



	AC	PAC	Q-Stat	Prob
1	-0.264	-0.264	8.0428	0.005
2	-0.013	-0.090	8.0637	0.018
3	-0.029	-0.062	8.1654	0.043
4	0.091	0.070	9.1434	0.058
5	-0.208	-0.182	14.283	0.014
6	0.125	0.031	16.151	0.013
7	0.015	0.043	16.177	0.024
8	0.046	0.063	16.439	0.037
9	0.066	0.142	16.980	0.049
10	0.034	0.063	17.123	0.072
11	-0.127	-0.073	19.174	0.058
12	0.021	-0.025	19.228	0.083
13	-0.033	-0.051	19.369	0.112
14	-0.166	-0.206	22.950	0.061
15	0.153	0.059	26.030	0.038
16	0.044	0.031	26.290	0.050
17	0.036	0.080	26.467	0.066
18	-0.097	-0.053	27.754	0.066
19	0.140	0.071	30.454	0.046
20	-0.062	0.086	30.981	0.055
21	-0.102	-0.083	32.427	0.053
22	-0.123	-0.178	34.582	0.043
23	0.181	0.069	39.257	0.019
24	-0.066	-0.020	39.896	0.022
25	-0.050	-0.156	40.268	0.027
26	0.065	0.017	40.892	0.032
27	0.109	0.115	42.684	0.028
28	-0.043	0.106	42.961	0.035
29	-0.076	-0.033	43.856	0.038
30	-0.095	-0.124	45.276	0.036
31	0.010	-0.013	45.292	0.047
32	0.030	-0.028	45.438	0.058
33	-0.079	-0.139	46.449	0.060
34	0.107	0.057	48.324	0.053
35	0.040	0.008	48.583	0.063

Fig. 5. Correlogram of DDNPER

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.369	-0.369	14.690	0.000
		2	0.086	-0.057	15.505	0.000
		3	-0.110	-0.113	16.838	0.001
		4	0.195	0.139	21.048	0.000
		5	-0.240	-0.144	27.537	0.000
		6	0.276	0.171	36.179	0.000
		7	-0.398	-0.302	54.369	0.000
		8	0.106	-0.171	55.665	0.000
		9	0.020	0.056	55.712	0.000
		10	0.056	-0.027	56.088	0.000
		11	-0.111	0.061	57.549	0.000
		12	0.045	-0.133	57.791	0.000
		13	-0.067	-0.016	58.333	0.000
		14	-0.104	-0.321	59.667	0.000
		15	0.194	0.088	64.352	0.000
		16	-0.096	0.008	65.504	0.000
		17	0.059	0.091	65.949	0.000
		18	-0.011	0.078	65.964	0.000
		19	0.060	-0.113	66.434	0.000
		20	-0.102	-0.025	67.798	0.000
		21	0.036	-0.348	67.967	0.000
		22	-0.179	-0.119	72.298	0.000
		23	0.211	0.167	78.402	0.000
		24	-0.101	0.096	79.831	0.000
		25	-0.026	-0.036	79.926	0.000
		26	0.079	0.012	80.804	0.000
		27	0.091	-0.007	81.990	0.000
		28	-0.046	-0.103	82.294	0.000
		29	-0.020	-0.114	82.354	0.000
		30	-0.080	-0.045	83.301	0.000
		31	-0.043	-0.038	83.577	0.000
		32	0.087	-0.054	84.735	0.000
		33	-0.059	-0.048	85.281	0.000
		34	0.082	0.141	86.334	0.000
		35	-0.009	0.034	86.348	0.000

Fig. 6. Correlogram of DSDDNPER

Table 2. ARIMA model estimation

Dependent Variable: DDNPER
Method: Least Squares
Date: 07/22/13 Time: 11:54
Sample(adjusted): 3 113
Included observations: 111 after adjusting endpoints
Convergence achieved after 11 iterations
Backcast: 2

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	0.068386	0.317327	0.215508	0.8298
MA(1)	-0.355051	0.297955	-1.191625	0.2360
R-squared	0.075855	Mean dependent var		-0.082802
Adjusted R-squared	0.067376	S.D. dependent var		1.327694
S.E. of regression	1.282186	Akaike info criterion		3.352864
Sum squared resid	179.1962	Schwarz criterion		3.401684
Log likelihood	-184.0839	F-statistic		8.946845
Durbin-Watson stat	1.981287	Prob(F-statistic)		0.003437
Inverted AR Roots	.07			
Inverted MA Roots	.36			

Table 3. SARIMA model estimation

Dependent Variable: DSDDNPER
Method: Least Squares
Date: 04/02/13 Time: 18:44
Sample(adjusted): 9 113
Included observations: 105 after adjusting endpoints
Convergence achieved after 16 iterations
Backcast: 1 8

Variable	Coefficient	Std. Error	t-Statistic	Prob.
MA(1)	-0.289903	0.081227	-3.569032	0.0005
MA(7)	-0.856073	0.000179	-4791.824	0.0000
MA(8)	0.348778	0.083825	4.160780	0.0001
R-squared	0.508658	Mean dependent var		-0.010410
Adjusted R-squared	0.499024	S.D. dependent var		1.865805
S.E. of regression	1.320611	Akaike info criterion		3.422221
Sum squared resid	177.8894	Schwarz criterion		3.498049
Log likelihood	-176.6666	F-statistic		52.79733
Durbin-Watson stat	1.978501	Prob(F-statistic)		0.000000
Inverted MA Roots	.95	.59+.77i	.59 -.77i	.41
	-.23+.96i	-.23 -.96i	-.89+.43i	-.89 -.43i

It is noteworthy that the β estimates are all statistically significant. As much as 51% of the variation in the data is explained by the model (4). The p-value of the regression is as low as .000000. There is close agreement between the fitted model and the data (Fig. 7). The correlogram of the residuals in Fig. 8 shows that they are uncorrelated. From the residual histogram in Fig. 9 it is evident that the probability curve of the residuals is dome-shaped and nearly normally distributed with zero mean.

On the contrary the ARIMA model (3) has statistically non-significant coefficients, a R^2 value as low as 8% as against 51% for the SARIMA model. The p-value of the regression is .003437 higher than that of the SARIMA model. Even though with a p-value of .008581 the hypothesis of a Gaussian SARIMA residual distribution is rejected (Fig. 9), the situation is worse for the ARIMA model with a p-value of 0.005911. The inferiority of the ARIMA model therefore is not in doubt.

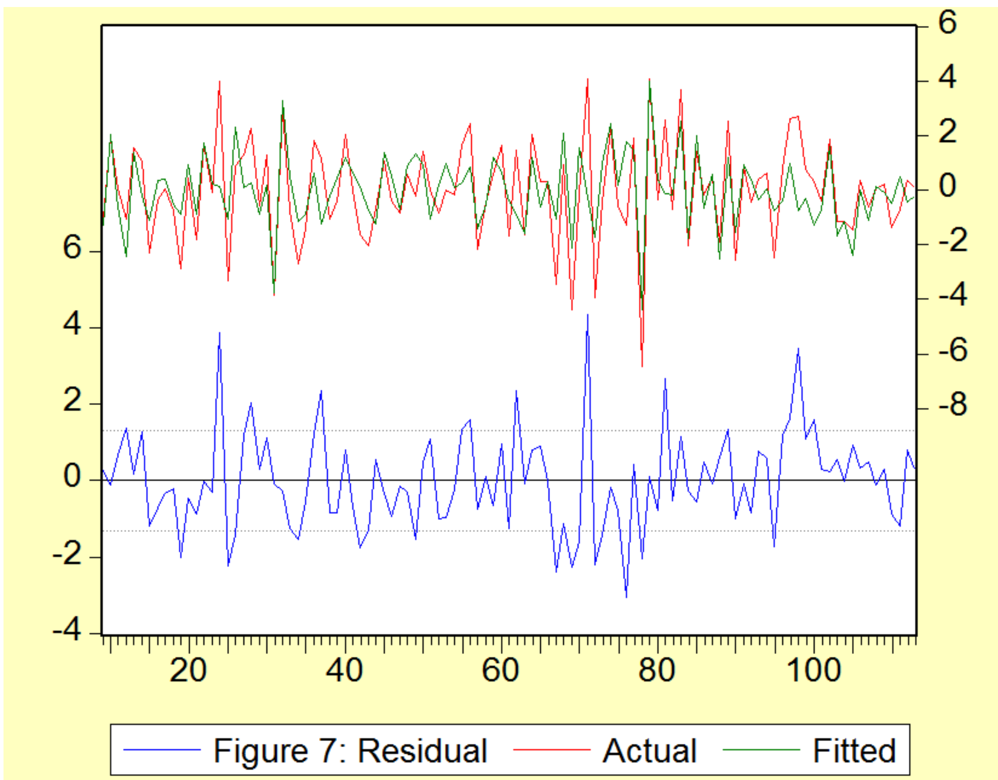
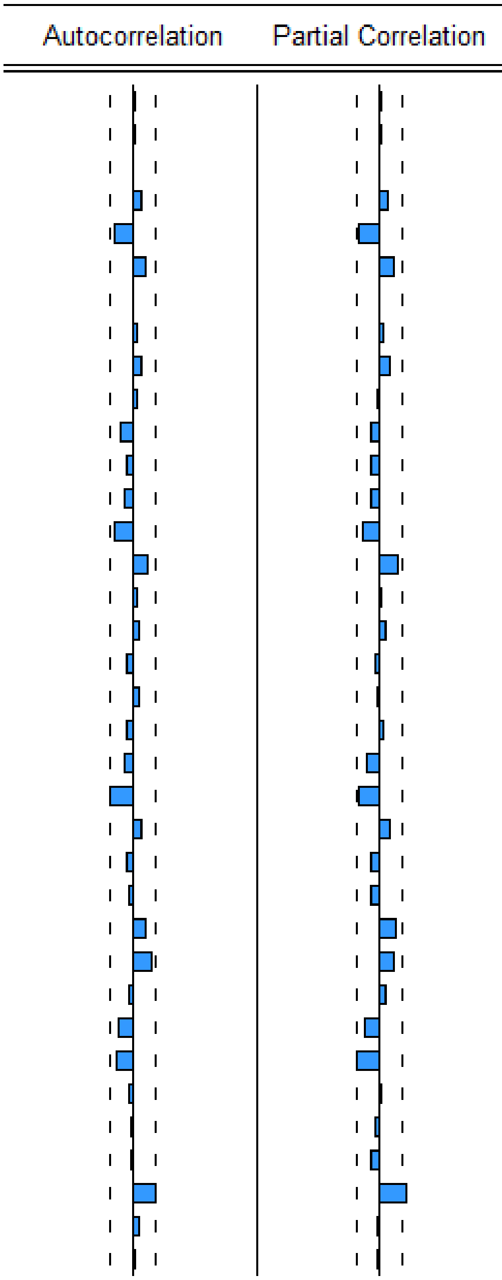


Figure 7: Residual



	AC	PAC	Q-Stat	Prob
1	0.010	0.010	0.0115	
2	0.021	0.021	0.0614	
3	-0.008	-0.008	0.0678	
4	0.071	0.071	0.6328	0.426
5	-0.164	-0.166	3.6476	0.161
6	0.111	0.118	5.0497	0.168
7	0.001	0.002	5.0499	0.282
8	0.035	0.025	5.1908	0.393
9	0.062	0.089	5.6443	0.464
10	0.041	-0.010	5.8393	0.559
11	-0.102	-0.071	7.0884	0.527
12	-0.055	-0.069	7.4574	0.590
13	-0.075	-0.075	8.1388	0.615
14	-0.153	-0.142	11.025	0.441
15	0.124	0.148	12.952	0.373
16	0.038	0.008	13.130	0.438
17	0.043	0.051	13.369	0.498
18	-0.042	-0.038	13.592	0.557
19	0.042	-0.009	13.825	0.612
20	-0.049	0.038	14.143	0.657
21	-0.079	-0.103	14.968	0.664
22	-0.189	-0.172	19.796	0.407
23	0.081	0.085	20.693	0.415
24	-0.042	-0.062	20.932	0.463
25	-0.027	-0.075	21.037	0.518
26	0.107	0.147	22.668	0.480
27	0.157	0.116	26.228	0.342
28	-0.030	0.043	26.362	0.388
29	-0.126	-0.126	28.720	0.324
30	-0.139	-0.190	31.615	0.247
31	-0.041	0.017	31.870	0.280
32	-0.011	-0.033	31.889	0.325
33	-0.011	-0.071	31.908	0.372
34	0.185	0.237	37.317	0.201
35	0.045	-0.023	37.646	0.227
36	0.022	-0.017	37.724	0.262

Fig. 8. Correlogram of the SARIMA residuals

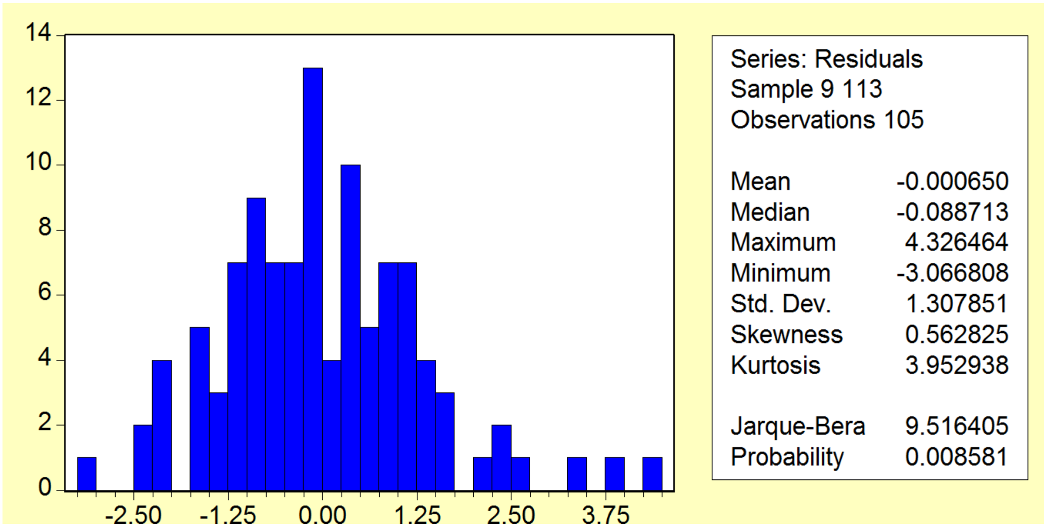


Fig. 9. Histogram of the SARIMA residuals

4. CONCLUSION

It is observed that the SARIMA model is better able to capture the intrinsically seasonal nature of the series DNPER than the ARIMA model. It may therefore be concluded that daily Naira-Pound exchange rates are seasonal of one-week period and follow a $(0, 1, 1)x(0, 1, 1)_7$ SARIMA model. This model has been shown to be relatively adequate.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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