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PULSE PROPAGATION THROUGH A MEDIUM CONSISTS OF TWO THREE-LEVEL ATOMS

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ABSTRACT

The coherent propagation of two optical short pulses through a resonant medium consisting of two three-level atoms in the Ξ configuration is investigated. A self-consisting analytical solution without steady state or adiabatic approximations is presented. The electric field in the model studied consists of two co-propagating plane waves, each of which is in near resonance with a transition in the absorber. The density matrix of two three level atoms is studied. Also, the reduced density matrices are stated. The present approach for the semiclassical treatment of resonant coherent interactions in three-level atoms represents a generalization of the Maxwell- Bloch equations for a two-level system. The Maxwell-Bloch equations reduced to the non-linear pendulum equation. The solution shows that the two pulses can propagate simultaneously without loss under some conditions about the pulses and the medium. The propagation of the two pulses through the medium without any constrains approves the self -induced transparency (SIT) phenomenon.

Keywords: Pulse propagation; Semi-classical approach; Maxwell Bloch equations; Non-linear equations; Density matrix .

1. INTRODUCTION

During the 18th century, different theories have been developed to describe light, the most well-known of which are the wave theory and the particle theory [1] . Now, scientists know that light can be viewed either as a wave" described by the classical electromagnetic wave theory, or as "a bunch of photons " quantized electromagnetic field described by quantum theory. Although the understanding of light has advanced significantly, there is still ongoing efforts to understand light itself as well as its interaction with different materials.

One aspect of research effort is devoted to the study of propagation of short light pulses in a variety of optical media [2]. Unlike the continuous light waves, a light pulse has a finite duration, as well as a time-dependent intensity profile. Such a unique feature makes light pulses very attractive, thus it is very important to investigate and understand the propagation behavior of light pulses.

The interaction of strong electromagnetic fields with atomic systems leads to nonlinear dynamics, which makes it difficult to solve analytically, but it is worth the effort. In the presence of two monochromatic lasers (or Cook-Shore pulses [3]), and under two-photon resonant conditions, the three level system has the eight-dimensional space which can be factorized into three independent subspaces. The quantum evolution of the three-level system is thus characterized by three independent coherence vectors, each with its own nonlinear conservation law [4].

The three-level atomic system has been studied in detail by many groups in quantum optics [5], non-linear optics and laser physics. In the theory of lasers a number of interesting and potentially important effects have been investigated, including, Non-Markovian decay [6], dynamics of a two-photon laser with an injected signal [7]-[10] and lasing without inversion. The numerical solutions show evidence of pulse evolution and/or breakups, possibly indicative of some global relationships governing the propagation problem at hand. Experimentally the three-level cascade atomic system has been studied for polarization effects in Rubidium [11] and laser cooling and diffusion in metastable Helium [12].

Here we study a medium of two species of three-level atoms interacting with classical field applying semi-classical approach. Ignoring the rapid time dependent factors in the Hamiltonian and the density matrix, the density matrix and the reduced density matrix are studied. Combined numerical and analytic techniques are used to solve the Maxwell-Bloch equations where we transform the system to the Non-linear pendulum equation.

2. THEORETICAL MODEL

In this paper we investigate the propagation of two intense optical beams in a medium of two species of three-level atoms in the Ξ configuration as shown in Fig. (1). The wave function for the atomic system can be written in the following form:

$$|\psi\rangle = |\psi\rangle_A \otimes |\psi\rangle_B \tag{1}$$

$$|\psi\rangle_A = c_\alpha e^{-i\omega_\alpha t}|\alpha\rangle + c_\beta e^{-i\omega_\beta t}|\beta\rangle + c_\gamma e^{-i\omega_\gamma t}|\gamma\rangle \tag{2}$$

$$|\psi\rangle_B = c_a e^{-i\omega_a t}|a\rangle + c_b e^{-i\omega_b t}|b\rangle + c_c e^{-i\omega_c t}|c\rangle, \tag{3}$$

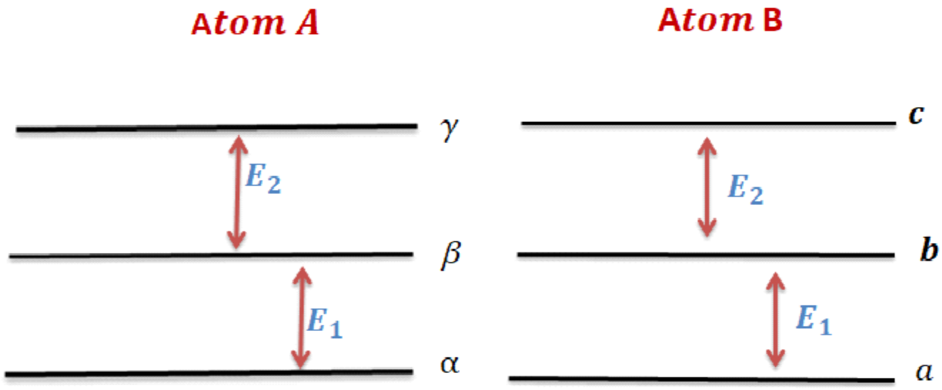

Fig. 1. Schematic diagram of two three-level $\Xi - type$ atoms interacting with classical fields.

The Hamiltonian which describes the interaction between two three-level atoms and electromagnetic fields is given by

$$\begin{aligned}\hat{\mathcal{H}} &= \hat{\mathcal{H}}_{atom} + \hat{\mathcal{H}}_I \\ &= \hat{\mathcal{H}}_{atom} - \vec{d}.\vec{E}(z,t)\end{aligned} \tag{4}$$

where the $\hat{\mathcal{H}}_{atom}$ is the atomic Hamiltonian in the absence of light which represented by the diagonal elements of $9*9$ matrix in the form

$\hbar\varpi_{ij}$ $(i = a, b, c), (j = \alpha, \beta, \gamma)$ where $\varpi_{ij} = \omega_i + \omega_j$, (ω_i, ω_j) are the frequencies of the energy levels. The electric field $\vec{E}(z,t)$ of the light is composed of two plane wave copropagating in the z-axis (with possible different velocities).

$$\vec{E}(z,t) = \sum_{i=1}^{2} \varepsilon_i(z,t)\cos\Phi_i\vec{e}_i \tag{5}$$

$$\Phi_i = \nu_i t - k_i z + \phi_i(z,t) \tag{6}$$

where $\varepsilon_i(z,t)$ are the amplitudes of the two waves, ν_i are the carrier frequencies, k_i denotes the wave number ($k_i = \frac{\nu_i}{c}$) and $\vec{e}_i$ are the unit polarization vectors. The two allowed atomic transitions are only between the excited and the intermediate states and between the intermediate and ground states. Both the amplitude and phase function are assumed to be slowly varying function of space and time so that,

$$\begin{aligned} &\left|\frac{\partial\varepsilon_i}{\partial z}\right| << k_i\varepsilon_i, \quad \left|\frac{\partial\varepsilon_i}{\partial t}\right| << \nu_i\varepsilon_i \\ &\left|\frac{\partial\phi_i}{\partial z}\right| << k_i\phi_i, \quad \left|\frac{\partial\phi_i}{\partial t}\right| << \nu_i\phi_i \end{aligned} \tag{7}$$

The electric dipole moment operator $\vec{d}$ is defined as

$$\vec{d} = \vec{d}_{\alpha\beta}|\alpha\rangle\langle\beta| + \vec{d}_{\beta\gamma}|\beta\rangle\langle\gamma| + \vec{d}_{ab}|a\rangle\langle b| + +\vec{d}_{bc}|b\rangle\langle c| + c.c \tag{8}$$

$$\begin{aligned} \vec{d}.\vec{E} \quad &= d_{\alpha\beta}\varepsilon_1^{*}e^{i\Phi_1}|\alpha\rangle\langle\beta| + d_{\beta\alpha}\varepsilon_1 e^{-i\Phi_1}|\beta\rangle\langle\alpha| \\ &+d_{ab}\varepsilon_1^{*}e^{i\Phi_1}|a\rangle\langle b| + d_{ba}\varepsilon_1 e^{-i\Phi_1}|b\rangle\langle a| \\ &+d_{\beta\gamma}\varepsilon_2^{*}e^{i\Phi_2}|\beta\rangle\langle\gamma| + d_{\gamma\beta}\varepsilon_2 e^{-i\Phi_2}|\gamma\rangle\langle\beta| \\ &+d_{bc}\varepsilon_2^{*}e^{i\Phi_2}|b\rangle\langle c| + d_{cb}\varepsilon_2 e^{-i\Phi_2}|c\rangle\langle b| \end{aligned} \tag{9}$$

The first step is to remove the rapid time dependence from the Hamiltonian (4) by using the transformation

$$U(t) = e^{-i\Lambda} \tag{10}$$

where Λ is a 9×9 diagonal matrix, its elements are: $\Lambda_{11} = \hbar\varpi_{a\alpha}$, $\Lambda_{22} = \hbar\varpi_{a\alpha} + \Phi_1$, $\Lambda_{33} = \hbar\varpi_{a\alpha} + \Phi_1 + \Phi_2$, $\Lambda_{44} = \hbar\varpi_{a\alpha} + \Phi_1$, $\Lambda_{55} = \hbar\varpi_{a\alpha} + 2\Phi_1$, $\Lambda_{66} = \hbar\varpi_{a\alpha} + 2\Phi_1 + \Phi_2$, $\Lambda_{77} = \hbar\varpi_{a\alpha} + \Phi_1 + \Phi_2$, $\Lambda_{88} = \hbar\varpi_{a\alpha} + 2\Phi_1 + \Phi_2$ and $\Lambda_{99} = \hbar\varpi_{a\alpha} + 2\Phi_1 + 2\Phi_2$.

Hence the Hamiltonian (4) will transform into a rotating frame [13] in which the Hamiltonian varies slowly in time as follows:

$$\hat{H} = U\hat{\mathcal{H}}U^{-1} + \hbar\frac{\partial\Lambda}{\partial t} \tag{11}$$

where, now the atomic Hamiltonian can be written in the form of the diagonal elements of the $9*9$ matrix respectively as follows:

$\hbar\varpi_{\alpha\alpha}$, $\hbar(\varpi_{\alpha\beta}+\wp_1)$, $\hbar(\varpi_{\alpha\gamma}+\wp_1+\wp_2)$, $(\hbar\varpi_{b\alpha}+\wp_1)$,
$\hbar(\varpi_{b\beta}+2\wp_1)$, $\hbar(\varpi_{b\gamma}+2\wp_1+\wp_2)$, $(\hbar\varpi_{c\alpha}+\wp_1+\wp_2)$, $\hbar(\varpi_{c\beta}+2\wp_1+\wp_2)$
and $\hbar(\varpi_{c\gamma}+2\wp_1+2\wp_2)$
where

$$\wp_i = \nu_i + \frac{\partial\phi_i}{\partial t} \tag{12}$$

and the interaction Hamiltonian will be

$$\hat{H}_I = \vec{d}.\vec{E} = \frac{\hbar}{2}\left[\Omega_{\alpha\beta}|\alpha\rangle\langle\beta| + \Omega_{\beta\gamma}|\beta\rangle\langle\gamma| + \Omega_{ab}|a\rangle\langle b| + \Omega_{bc}|c\rangle\langle b| + c.c\right] \tag{13}$$

where

$$\begin{aligned} \Omega_{\alpha\beta} &= \frac{2}{\hbar}(\vec{d}_{\alpha\beta}.\vec{e}_1)\varepsilon_1^*(z,t), \qquad & \Omega_{\beta\gamma} &= \frac{2}{\hbar}(\vec{d}_{\beta\gamma}.\vec{e}_2)\varepsilon_2^*(z,t), \\ \Omega_{ab} &= \frac{2}{\hbar}(\vec{d}_{ab}.\vec{e}_1)\varepsilon_1^*(z,t), \qquad & \Omega_{bc} &= \frac{2}{\hbar}(\vec{d}_{bc}.\vec{e}_2)\varepsilon_2^*(z,t) \end{aligned} \tag{14}$$

2.1. Density Matrix Formalism

Because of the density matrix can describes a wider range of atomic behavior than the wave function, we specify the atomic system by the density matrix $\hat{\rho}$, where $\hat{\rho} = |\psi\rangle\langle\psi|$ and from (10)

$$\hat{\rho} = U(t)\rho U^{-1}(t) \tag{15}$$

The reduced density operator for the atom with levels $\alpha\beta\gamma$ is $\hat{\rho}_A$ and for the atom with levels abc is $\hat{\rho}_B$ can be written as

$$\begin{aligned} \hat{\rho}_A = Tr_B\hat{\rho} &= \begin{pmatrix} \hat{\rho}_{11}+\hat{\rho}_{44}+\hat{\rho}_{77} & \hat{\rho}_{12}+\hat{\rho}_{45}+\hat{\rho}_{78} & \hat{\rho}_{13}+\hat{\rho}_{46}+\hat{\rho}_{79} \\ \hat{\rho}_{21}+\hat{\rho}_{54}+\hat{\rho}_{87} & \hat{\rho}_{22}+\hat{\rho}_{55}+\hat{\rho}_{88} & \hat{\rho}_{47}+\hat{\rho}_{58}+\hat{\rho}_{69} \\ \hat{\rho}_{31}+\hat{\rho}_{64}+\hat{\rho}_{97} & \hat{\rho}_{32}+\hat{\rho}_{56}+\hat{\rho}_{98} & \hat{\rho}_{33}+\hat{\rho}_{66}+\hat{\rho}_{99} \end{pmatrix} \\ &= \begin{pmatrix} \hat{\rho}_{11}^{(A)} & \hat{\rho}_{12}^{(A)} & \hat{\rho}_{13}^{(A)} \\ \hat{\rho}_{21}^{(A)} & \hat{\rho}_{22}^{(A)} & \hat{\rho}_{23}^{(A)} \\ \hat{\rho}_{31}^{(A)} & \hat{\rho}_{32}^{(A)} & \hat{\rho}_{33}^{(A)} \end{pmatrix} \end{aligned} \tag{16}$$

where A is related to the first atom. Also,

$$\begin{aligned} \hat{\rho}_B = Tr_A\hat{\rho} &= \begin{pmatrix} \hat{\rho}_{11}+\hat{\rho}_{22}+\hat{\rho}_{33} & \hat{\rho}_{14}+\hat{\rho}_{25}+\hat{\rho}_{36} & \hat{\rho}_{17}+\hat{\rho}_{28}+\hat{\rho}_{39} \\ \hat{\rho}_{41}+\hat{\rho}_{52}+\hat{\rho}_{63} & \hat{\rho}_{44}+\hat{\rho}_{55}+\hat{\rho}_{66} & \hat{\rho}_{47}+\hat{\rho}_{58}+\hat{\rho}_{69} \\ \hat{\rho}_{71}+\hat{\rho}_{82}+\hat{\rho}_{93} & \hat{\rho}_{74}+\hat{\rho}_{85}+\hat{\rho}_{96} & \hat{\rho}_{77}+\hat{\rho}_{88}+\hat{\rho}_{99} \end{pmatrix} \\ &= \begin{pmatrix} \hat{\rho}_{11}^{(B)} & \hat{\rho}_{12}^{(B)} & \hat{\rho}_{13}^{(B)} \\ \hat{\rho}_{21}^{(B)} & \hat{\rho}_{22}^{(B)} & \hat{\rho}_{23}^{(B)} \\ \hat{\rho}_{31}^{(B)} & \hat{\rho}_{32}^{(B)} & \hat{\rho}_{33}^{(B)} \end{pmatrix} \end{aligned} \tag{17}$$

where B is related to the second atom.

Subsequently, we can write the propagation equation for these slowly varying density matrix elements [14, 15] as

$$\hat{\rho}^{\cdot} = -\frac{i}{\hbar}[\hat{H}, \hat{\rho}] \tag{18}$$

where $\hat{\rho}_{12}^{(i)} = \rho_{12}^{(i)} e^{-i(\wp_1 t - k_1 z)}$, $\hat{\rho}_{23}^{(i)} = \rho_{23}^{(i)} e^{-i(\wp_2 t - k_2 z)}$,

$\hat{\rho}_{13}^{(i)} = \rho_{13}^{(i)} e^{-i((\wp_1+\wp_2)t-(k_1+k_2)z)}$ and $\hat{\rho}_{jj}^{(i)} = \rho_{jj}^{(i)}$,

$(i = A, B, \qquad j = 1,2,3)$.

2.2. The Equations of Motion

The equations of motion for the components of the density matrix vector in the rotating-wave approximation are

$$\begin{aligned}
\hat{\rho}_{12}^{\cdot(A)} &= -i\Delta_1\hat{\rho}_{12}^{(A)} + \frac{i}{2}\Omega_{\beta\gamma}\hat{\rho}_{13}^{(A)} + \frac{i}{2}\Omega_{\beta\alpha}(\hat{\rho}_{11}^{(A)} - \hat{\rho}_{22}^{(A)})\\
\hat{\rho}_{12}^{\cdot(B)} &= -i\Delta_1\hat{\rho}_{12}^{(B)} + \frac{i}{2}\Omega_{bc}\hat{\rho}_{13}^{(B)} + \frac{i}{2}\Omega_{ba}(\hat{\rho}_{11}^{(B)} - \hat{\rho}_{22}^{(B)})\\
\hat{\rho}_{23}^{\cdot(A)} &= -i\Delta_2\hat{\rho}_{23}^{(A)} - \frac{i}{2}\Omega_{\alpha\beta}\hat{\rho}_{13}^{(A)} - \frac{i}{2}\Omega_{\gamma\beta}(\hat{\rho}_{33}^{(A)} - \hat{\rho}_{22}^{(A)})\\
\hat{\rho}_{23}^{\cdot(B)} &= -i\Delta_2\hat{\rho}_{23}^{(B)} - \frac{i}{2}\Omega_{ab}\hat{\rho}_{13}^{(B)} - \frac{i}{2}\Omega_{cb}(\hat{\rho}_{33}^{(B)} - \hat{\rho}_{22}^{(B)})\\
\hat{\rho}_{13}^{\cdot(A)} &= -i(\Delta_1 + \Delta_2)\hat{\rho}_{13}^{(A)} + \frac{i}{2}\Omega_{\gamma\beta}\hat{\rho}_{12}^{(A)} - \frac{i}{2}\Omega_{\beta\alpha}\hat{\rho}_{23}^{(A)}\\
\hat{\rho}_{13}^{\cdot(B)} &= -i(\Delta_1 + \Delta_2)\hat{\rho}_{13}^{(B)} + \frac{i}{2}\Omega_{cb}\hat{\rho}_{12}^{(B)} - \frac{i}{2}\Omega_{ba}\hat{\rho}_{23}^{(B)}
\end{aligned} \tag{19}$$

where $\omega_{ba} - \nu_1 = \Delta_1 = \omega_{\beta\alpha} - \nu_1, \qquad \omega_{cb} - \nu_2 = \Delta_2 = \omega_{\gamma\beta} - \nu_2$.

We can define the time dependent Rabi frequency Ω_{ij} in terms of its real and imaginary parts as follows:

$$\begin{aligned}
\Omega_{\alpha\beta} &= R_{\alpha\beta} + iU_{\alpha\beta} = \Omega_{\beta\alpha}^*\\
\Omega_{\beta\gamma} &= R_{\beta\gamma} + iU_{\beta\gamma} = \Omega_{\gamma\beta}^*\\
\Omega_{ab} &= R_{ab} + iU_{ab} = \Omega_{ba}^*\\
\Omega_{bc} &= R_{bc} + iU_{bc} = \Omega_{cb}^*
\end{aligned} \tag{20}$$

At this stage we will restrict ourselves to Rabi frequencies to be real and the two atoms to be identical, so we can write

$$\begin{aligned}
R_{\alpha\beta} &= R_{12} = R_{ab}\\
R_{\beta\gamma} &= R_{23} = R_{bc},
\end{aligned} \tag{21}$$

and $\vec{d}_{\alpha\beta} = \vec{d}_{ab} = \vec{d}_{12}, \qquad \vec{d}_{\alpha\beta} = \vec{d}_{bc} = \vec{d}_{23}$.

The components of the Bloch vector are defined as

$$\begin{aligned}
u_{ij}^{(l)} &= \hat{\rho}_{ij}^{(l)} + \hat{\rho}_{ji}^{(l)}\\
v_{ij}^{(l)} &= i(\hat{\rho}_{ij}^{(l)} - \hat{\rho}_{ji}^{(l)})
\end{aligned} \tag{22}$$

so that the equations of motion for the components of the generalized Bloch vectors and the atomic population in the exact resonance case can be obtained by differentiate (22) and substitute from (19), taking into account (20, 21) as:

$$\begin{aligned}
\dot{u}_{12}^{(l)} &= \frac{1}{2}R_{23}v_{13}^{(l)} \\
\dot{v}_{12}^{(l)} &= R_{12}\omega_{21}^{(l)} - \frac{1}{2}R_{23}u_{13}^{(l)} \\
\dot{u}_{23}^{(l)} &= -\frac{1}{2}R_{12}v_{13}^{(l)} \\
\dot{v}_{23}^{(l)} &= R_{23}\omega_{32}^{(l)} + \frac{1}{2}R_{12}u_{13}^{(l)} \\
\dot{u}_{13}^{(l)} &= -\frac{1}{2}R_{12}v_{23}^{(l)} + \frac{1}{2}R_{23}v_{12}^{(l)} \\
\dot{v}_{13}^{(l)} &= \frac{1}{2}R_{12}u_{23}^{(l)} - \frac{1}{2}R_{23}u_{12}^{(l)} \\
\dot{\hat{\rho}}_{11}^{(l)} &= \frac{1}{2}R_{12}v_{12}^{(l)} \\
\dot{\hat{\rho}}_{22}^{(l)} &= -\frac{1}{2}R_{12}v_{12}^{(l)} + \frac{1}{2}R_{23}v_{23}^{(l)} \\
\dot{\hat{\rho}}_{33}^{(l)} &= -\frac{1}{2}R_{23}v_{23}^{(l)}.
\end{aligned} \tag{23}$$

where $w_{21}^{(l)} = \hat{\rho}_{22}^{(l)} - \hat{\rho}_{11}^{(l)}$ and $w_{32}^{(l)} = \hat{\rho}_{33}^{(l)} - \hat{\rho}_{22}^{(l)}, l = A, B$

It is possible to obtain from (23) the following conservation law

$$\begin{aligned}
&(v_{12}^{(l)})^2 + (u_{12}^{(l)})^2 + (v_{23}^{(l)})^2 + (u_{23}^{(l)})^2 + (v_{13}^{(l)})^2 + (u_{13}^{(l)})^2 \\
&+2[(\hat{\rho}_{11}^{(l)})^2 + (\hat{\rho}_{22}^{(l)})^2 + (\hat{\rho}_{33}^{(l)})^2] = C
\end{aligned} \tag{24}$$

where C is a constant. For one atom the conservation law is similar to the one [16] which reminiscent a single two-level atom.

2.3. The Maxwell-Bloch Equation

The atom-field interaction can be described through Maxwell's equation [2]. It is sufficient to write it in one dimensional form:

$$\left(\frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{E}(t,z) = \frac{4\pi}{c^2}\frac{\partial^2}{\partial t^2}\vec{P}(z,t) \tag{25}$$

where $\vec{P}$ is the (space and time dependent) polarization. If we consider that the two fields interact with a homogeneously distributed two three-level atomic system in the cascade configurations then, the polarization induced by two fields will be the average dipole moment per unit volume and can be defined as

$$\begin{aligned}
\vec{P}(z,t) &= \vec{P}_1(z,t) + \vec{P}_2(z,t) \\
&= N\sum_l Tr(\hat{\rho}_{ij}^{(l)}\vec{d}_{ij}), \qquad (l = A, B),
\end{aligned} \tag{26}$$

where N is the atomic density. Using Eqs. (14, 20, 21 and 22) so, Maxwell's equation in the slowly varying envelope approximation (SVEA) can be simplified (considering the real part of the Rabi frequencies only) to the form

$$\begin{aligned}
\left(\frac{\partial}{\partial z} + \frac{\partial}{\partial ct}\right)R_{12} &= \frac{1}{2}G_a(v_{12}^{(A)} + v_{12}^{(B)}) \\
\left(\frac{\partial}{\partial z} + \frac{\partial}{\partial ct}\right)R_{23} &= \frac{1}{2}G_a\beta(v_{23}^{(A)} + v_{23}^{(B)})
\end{aligned} \tag{27}$$

Where $G_a = \frac{4N\pi\wp_1|d_{12}|^2}{\hbar c}$, $\beta = \frac{\wp_2|d_{23}|^2}{\wp_1|d_{12}|^2}$. When we chose the two atoms to be identical, in this case $v_{12}^{(l)} = v_{12}$, $v_{23}^{(l)} = v_{23}$ so, Eq. (27) can be written as

$$(\frac{\partial}{\partial z}+\frac{\partial}{\partial ct})R_{12}=G_a v_{12}$$
$$(\frac{\partial}{\partial z}+\frac{\partial}{\partial ct})R_{23}=G_a \beta v_{23} \tag{28}$$

Now, we set

$$v_{12}=b_1\sin\sigma, \qquad v_{23}=b_2\sin\sigma \tag{29}$$

with $\sigma=\int_{-\infty}^{t}\Omega(z,`t)d`t$, hence $\Omega(z,t)=\frac{\partial\sigma}{\partial t}$. We define

$$R_{12}=\alpha_1\Omega, \qquad R_{23}=\alpha_2\Omega, \tag{30}$$

where α_1, α_2 are the relative amplitude of the pulses, Eqs. (30) define the pulses as generalized Cook-Shore pulses [3].

Then Eq. (28) reduces to

$$\frac{\partial^2\sigma}{\partial t\partial z}+\frac{1}{c}\frac{\partial^2\sigma}{\partial t^2}=y\sin\sigma \tag{31}$$

where

$$y=\frac{G_a b_1}{\alpha_1}=\frac{G_a b_2\beta}{\alpha_2} \tag{32}$$

The distortionless solution of Eq. (31) has the argument $\xi=t-\frac{z}{V}$ where V has dimensions of velocity. Then $\frac{\partial}{\partial z}=-\frac{1}{V}\frac{\partial}{\partial t}$, so that Eq. (31) becomes:

$$\frac{\partial^2\sigma}{\partial t^2}=-[yc(\frac{c}{V}-1)^{-1}]\sin\sigma \tag{33}$$

This equation is similar to that given in [17], so the solution of this non-linear pendulum equation is

$$\sigma=2\sin^{-1}(\tanh(\frac{\xi}{\tau})) \tag{34}$$

where τ is the pulse length and is define as

$$\frac{1}{\tau^2}=\frac{G_a b_1}{\alpha_1(\frac{1}{V}-\frac{1}{c})}=\frac{G_a b_2}{\alpha_2(\frac{1}{V}-\frac{1}{c})}, \tag{35}$$

Consequently, from (34)

$$\Omega=\frac{2}{\tau}sech(\frac{\xi}{\tau}) \tag{36}$$

and from (29), we get

$$\begin{aligned} v_{12}(\xi)&=b_1 sech(\frac{\xi}{\tau})\tanh(\frac{\xi}{\tau}),\\ v_{23}(\xi)&=b_2 sech(\frac{\xi}{\tau})\tanh(\frac{\xi}{\tau})\\ w_{21}&=b_3+b_4 sech^2(\frac{\xi}{\tau}),\\ w_{32}&=b_5+b_6 sech^2(\frac{\xi}{\tau}),\\ u_{13}&=b_7 sech^2(\frac{\xi}{\tau}) \end{aligned} \tag{37}$$

Where the constants b_3 and b_5 are the initial inversions $w_{21}(\xi = -\infty)$ and $w_{32}(\xi = -\infty)$ respectively. From the system (37) we can define the constants b_i, $i = 1,2,4,5,6,7$ in terms of b_3 as follows:

$$b_1 = -2\alpha_1 b_3, \quad b_2 = -2\frac{\alpha_1^2}{\alpha_2} b_3, \quad b_4 = -\alpha_1^2 b_3 = a_6, \quad b_5 = \frac{\alpha_1^2}{\alpha_2^2} b_3, \quad and \quad b_7 = \alpha_1\alpha_2\left(1 - \frac{\alpha_1^2}{\alpha_2^2}\right)b_3. \tag{38}$$

with the condition

$$\alpha_1^2 + \alpha_2^2 = 4 \tag{39}$$

Using Eqs.(28,37 and 38) we obtain

$$\frac{1}{\tau^2} = -\frac{G_a b_3}{(\frac{1}{V} - \frac{1}{c})} \tag{40}$$

and

$$\frac{\alpha_2^2}{\beta\alpha_1^2} = 1 \tag{41}$$

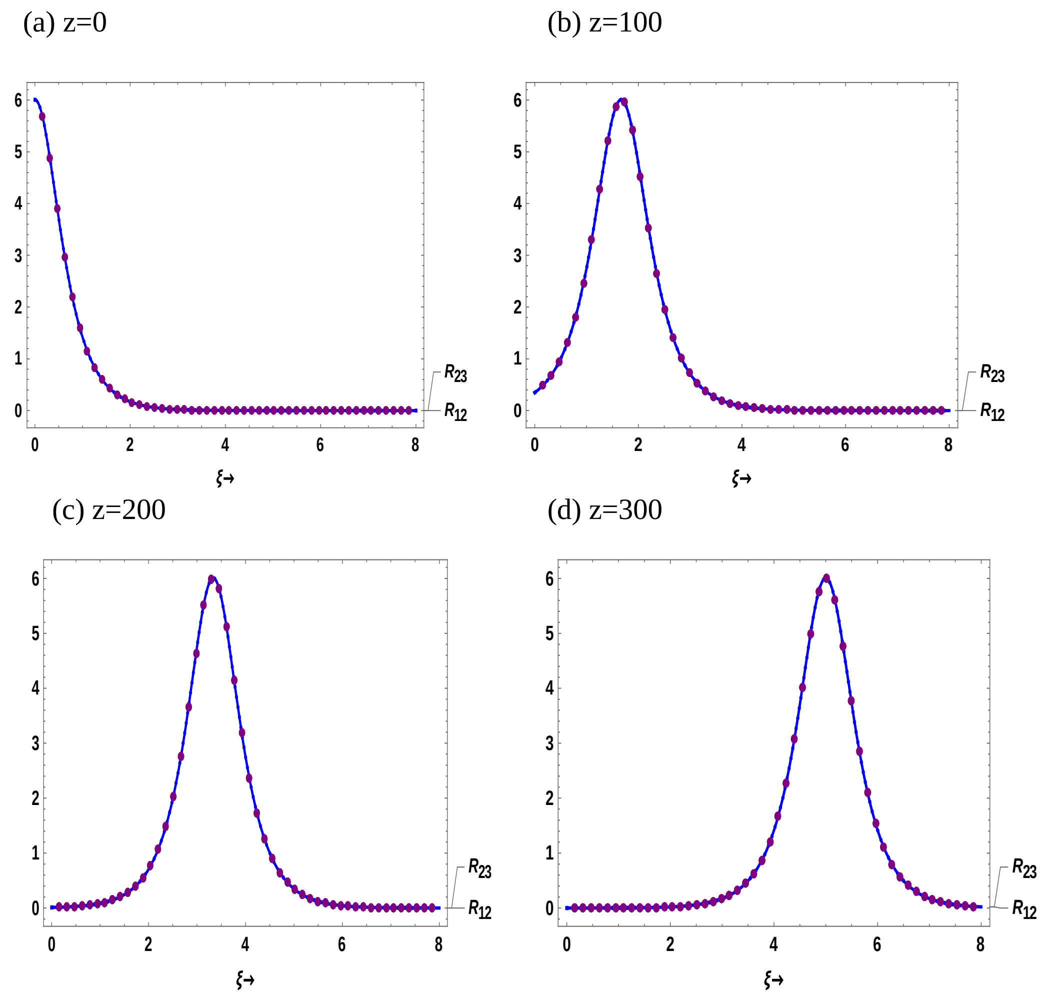

Fig. 2. The temporal behavior of the pulse profile at different values of the probagation distance z.

In order to have $\tau^2 > 0$, since $V < c$, we have to require that $b_3 < 0$. We obtain the solutions to the full set of on-resonance two three-level Maxwell-Bloch equations. These have the characteristics of simultaneous different-wavelength optical solitons. In the Ξ −medium the profile (30) propagate as simultaneous pulses at a certain conditions, imposed on the initial preparation of both the pulses and the medium, and determined by the medium's physical parameters. Eq. (41) is considered as a restriction on the relative amplitudes of the pulses.

For the three-level atom the simultaneous pulse propagation is not possible in a $\Xi-$ configuration in the case of the atoms totally prepared initially in the ground state i.e $b_3 = -1$, this only possible for an appropriate initial mixed states. These initial conditions are achieved by choosing the following initial populations , $\rho_{11} = \frac{2}{3}$, $\rho_{22} = \frac{1}{3}$ and $\rho_{33} = 0$. Addition to $\rho_{22} = const.$, through the entire interaction which has been derived not assumed. We should also note that the initial inversions are equal.

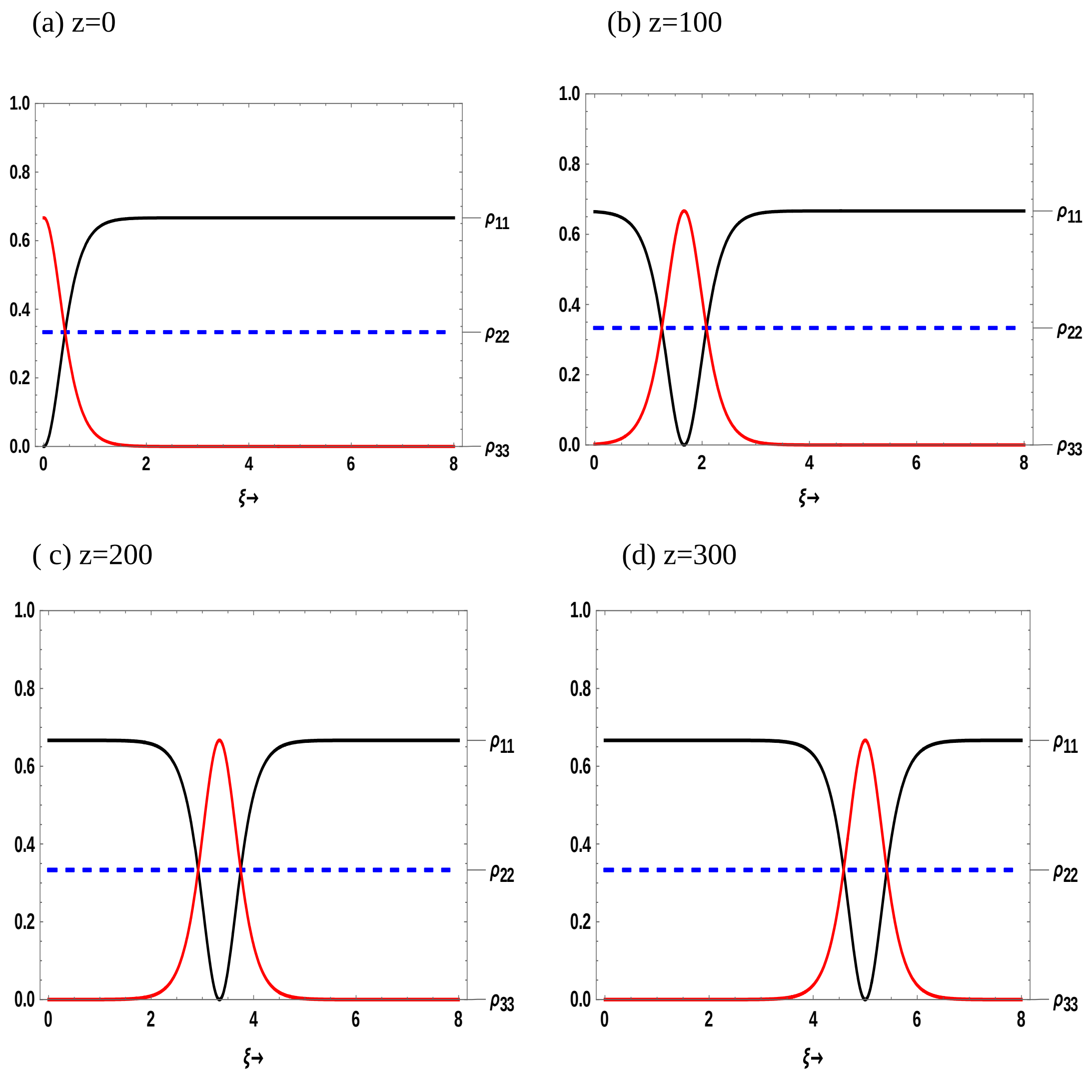

Fig. 3. The atomic occupation ρ_{11} black line, ρ_{22} blue line, ρ_{33} red line.

In Figs. (2,3) we plot the temporal behavior of the pulses and the level populations at the entry face of the medium ($z = 0$). The pulses are simultionous and identical: their peaks coincide, their pulse length is the same equal $\frac{\sqrt{2}}{3}$, which greater than the pulse length for one atom of three-level. The areas of each pulse is equal to $\sqrt{2}\,(2\pi)$. From Fig. (2) it is evident that the two input pulses propagate without loss and with shape preserving. Also the peak of the the two pulses is later in time at z position. In Fig. (3), the atomic dynamics represented by the three population (ρ_{11}, ρ_{22} and ρ_{33}). The behavior of them characterized by a full rotation of the initial state from $\rho_{11} = \frac{2}{3}$, $\rho_{22} = \frac{1}{3}$ and $\rho_{33} = 0$ to $\rho_{11} = 0$, $\rho_{22} = \frac{1}{3}$ and $\rho_{33} = \frac{2}{3}$, when the pulses are at their coincidence peaks (comparing between Fig. (2), (3)). The population of the second level (ρ_{22}) remains constant during the interaction, while the populations ρ_{11} and ρ_{33}.

3. CONCLUSION

We have introduced the Hamiltonian of the interaction between two three-level atoms and classical field. After using the approximation we calculated the density matrix and the reduced density matrix. We have solved Maxwell Bloch equations and reduced it to nonlinear pendulum equation. The solution represented pulse propagations differ with the pulse through a medium of one three-level atom in the amplitude and the pulse length. The possibility of simultinous lossless propagation of the two optical pulses is established. It turned out that the simultanous propagation occur when the pulse and the medium have to be prepared in a manner determined by the physical parameters. One recognizes that certain solution pulses propagation described as generalized Cook-Shore pulses since they have a common time dependence with possibly different amplitudes.

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REFERENCES

[1] Chen X. The Geometric tradition and the wave theory of light. l st ed. Springer Science + Business Media Dordrech. Kluwer Academic; (2000).
[2] Allen L, Eberly J H. Optical reasonance and two-level atom. 8th ed. John Weily, New York; 1987.
[3] Cook R J, Shore B. Coherent dynamics of N-level atoms and molecules. III. An analytically soluble periodic case. J Phys Rev A. 1979; 20 : 539. doi.org/10.1103/PhysRevA.20.539.
[4] Hioe F T, Eberly H J. Nonlinear constants of motion for three-level quantum systems. Phys Rev Lett . 1982; 4 : 25. doi.org/10.1103/PhysRevA.25.2168.
[5] Nath M R, Sen S. Dynamics of cascade three level system interacting with the classical and quantized field. Pramana J. Phys. 2003; 61 :1089. doi.org/10.1007/BF02704404.
[6] Dalton B J, Garraway BM. Non-Markovian decay of a three-level cascade atom in a structured reservoir, Phys. Rev. A. 2003; 68 : 33809. doi.org/10.1103/PhysRevA.68.033809.
[7] Shchedrin G, O'Brien C, Rostovtsev Y, Scully MO. Analytic solution and pulse area theorem for three-level atoms, Phys. Rev. A. 2015; 92 :063815. doi.org/10.1103/PhysRevA.92.063815.
[8] Grira S, Eleuch H. Solitary waves propagation in three-level atomic media. J. Phys. Conf. Ser. 2019; 1411: 012018. Doi:10.1088/1742-6596/1411/1/012018
[9] Groves E, Clader, Eberly JH. Jaynes–Cummings theory out of the box, J. Phys. B: At. Mol. Opt. Phys. 2013; 46 : 224005. https://doi.org/10.1088/0953-4075/46/22/224005.

[10] Urchueguia J F, Espinosa V, Valcarcel G.J. De. Nonlinear dynamics of a class-a two-photon laser with injected signal in cascade systems. J. Mod. Opt. 1999; 46 : 1483. https://doi.org/10.1080/09500349908231349.
[11] McGloin D, Dunn M H, Fulton D J. Polarization effects in electromagnetically induced transparency. Phys. Rev. A. 2000; 62 : 53802. doi.org/10.1103/PhysRevA.62.053802.
[12] Rooijakkers W, Hogervorst W, Vassen W. Laser cooling, friction, and diffusion in a three-level cascade system. Phys. Rev. A. 1997; 56 : 3083. doi.org/10.1103/PhysRevA.56.3083.
[13] Grischkowsky D, Loy M M T, Liao P F. Adiabatic following model for two-photon transitions: nonlinear mixing and pulse propagation. *Phys. Rev . A*. 1975, 12 : 2514. doi.org/10.1103/PhysRevA.12.2514.
[14] Scully M O, Zubairy M S. Quantum optics. 1st ed. United kingdom at University press, Cambredge; 1999.
[15] Meystre P. Semiclassical Atom–Light Interaction. *Quantum Optics*. Springer; 2021. doi.org/10.1007/978-3-030-76183-72021.
[16] Konopnicki M J, Eberly J H. Simultaneous propagation of short different-wavelength optical pulses, Phsyics A. 1981; 24 : 2567. doi.org/10.1103/PhysRevA.24.2567.
[17] Bullough R K, Caudrey P D, Eilbeck D C. A general theory of self-induced transparency, Opto-electronics. 1974; 6 : 121. doi.org/10.1007/BF01421994.

الملخص العربي

انتشار النبضة خلال وسط يتكون من ذرتين ذاتا ثلاث مستويات للطاقة

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الملخص:

قمنا بدراسة الانتشار المتلاحم لنبضتين خلال وسط يتكون من ذرتين كل منهما ذات ثلاث مستويات للطاقة في الوضع السلمي Ξ. و يتكون المجال الكهربي من موجتين مختلفتين في الطور . تم تقديم حل تحليلي لهذا النظام بدون استحدام أي تقريب . قمنا بحساب مصفوفة الكثافة لهذه المنظومة ثم اختزلناها الي مصفوفتين. ايضا قدمنا معادلات ماكسويل- بلوك والتي تعتبر في هذا التقريب الكلاسيكي للذرات ذي الثلاث مستويات هي حالة أعم من مثيلاتها للذرات ذات المستويين فقط. ثم اختزلنا معادلة ماكسويل - بلوك الي معادلة بندول غير خطي. استنتجنا من ذلك ان النبضتين تنتشران في وقت واحد دون تغيير تحت شروط عليهما وعلي الوسط ايضا. حيث أن إنتشار النبضات بدون قيود عبر الاوساط يوافق ظاهرة الشفافية المستحثة ذاتيا.